Robust full Stokes imaging polarimeter with dynamic calibration

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We present a full Stokes imaging polarimeter using a rotating retarder in combination with a polarization camera—a detector array on which a pixelated polarizer array is attached. By itself, a polarization camera cannot capture the full Stokes parameters, but we add a rotating retarder in front and show how it can be used to provide full Stokes images. In addition, we demonstrate the advantage that it can be recalibrated dynamically while taking measurements, allowing accurate measurements even in environments where the retardance is changing.

Polarization measurement is important in many applications, such as visualizing stress within a material and determining an object’s surface orientation and shape. The complete polarization state of incoherent light can be expressed using the four Stokes parameters, indicating the overall light intensity component, the difference between horizontal and vertical linear polarization, the difference between 45° and 135° linear polarization, and the difference between right and left circular polarization.

Many methods for Stokes parameter measurement have been proposed [1–7], among which one widely used method employs a rotating retarder and a rotating analyzer [1]. This method is capable of calibrating the waveplate retardance simultaneously with measurement of the Stokes parameters. It is difficult to operate at high speed due to the trouble of accurately synchronizing dual continuously rotating motors together with the camera readout at high speed. As a result, typical dual rotating retarder and analyzer systems designed for imaging generally require a few minutes for measurements.

Another polarization measurement method employs pairs of liquid crystal retarders or photoelastic modulators [2,3], allowing measurement without mechanical rotation. While these two methods can measure quickly, they possess other drawbacks. And whereas a liquid crystal polarimeter can often achieve images at up to 30 frames per second (fps), it is known to have a lower accuracy than other methods due to nonuniformity among the liquid crystal elements and scattering. Finally, a system based on photoelastic modulators can often achieve a rate limited by the camera readout speed, but it has a small aperture that limits light collection and field of view.

Stokes imaging can also be performed in snapshot form (i.e., at a rate limited by the detector readout) by encoding the polarization state into a channeled image, without electronic or mechanical scanning [4]. Its resolution, however, is reduced by the need to perform low-pass filtering of the image in order to reconstruct the Stokes images.

Polarization cameras—detector arrays to which have been attached pixelated micropolarizer filters (see Fig. 1)—have recently become available, allowing the imaging of three Stokes parameters, s0, s1, and s2, in snapshot form [5,6]. However, these cameras cannot measure s3 directly because the filter array does not include a retarder element. A polarization camera that does employ a microretarder array attached to a micropolarizer array and CCD sensor has been demonstrated [7]. This camera can measure the full Stokes parameters in real-time, but requires that the individual retardances in the microretarder array be calibrated in advance of a measurement. This can cause difficulty in the presence of temperature changes, which cause the retardances to change as well. Imaging the Stokes parameters with a rotating monolithic retarder and micropolarizer-equipped polarization camera allows a user to remove this temperature sensitivity.

This paper demonstrates two new methods for measuring the complete spatially resolved Stokes parameters, including s3, either statically or dynamically, using a single rotating retarder element placed in front of the polarization camera. Vaughn et al. proposed a Mueller matrix polarimeter using rotating retarders and a micropolarizer array but have not applied this approach to Stokes imaging and do not investigate its dynamic calibration capabilities [8]. In method 1, the retardance of the rotating retarder is calibrated in advance, and then Stokes parameters are measured. In method 2, the retardance of the rotating

![Fig. 1. Optical setup of rotating retarder and polarization camera.](#)
retarder is calibrated dynamically, at the same time as the Stokes parameter measurements. The latter method is particularly useful in environments where the retardance cannot be controlled, such as when measuring in the presence of changes in the temperature or the observation wavelength. Figure 1 shows the setup for Stokes parameter measurement, in which the micropolarizer elements have orientations $\varphi = 0^\circ$, $45^\circ$, $90^\circ$, and $135^\circ$ [5].

The Mueller calculus representing the polarized light propagation through the system is given by

$$S_{\text{out}} = A \cdot R \cdot S_{\text{in}},$$

where $S_{\text{in}}$ is the input polarization state of the light, $S_{\text{out}}$ the state after the polarizer elements and incident on CCD sensor, and $R$ and $A$ the Mueller matrices of the rotating retarder and analyzer. From this equation, we can obtain the detected intensity $I$ at the CCD sensor as

$$I = \frac{1}{2}(s_0 + [s_1\{1 - (1 - \cos \delta)\sin^2 2\xi\} + s_2(1 - \cos \delta)\sin 2\xi \cos 2\xi - s_3 \sin \delta \sin 2\xi] \cos 2\varphi$$

$$+ [s_1(1 - \cos \delta)\sin 2\xi \cos 2\xi + s_2(1 - (1 - \cos \delta)\cos^2 2\xi)] + s_3 \sin \delta \cos 2\xi \sin 2\varphi),$$

where $s_i$, $\delta$, and $\xi$ in Eq. (2) are elements of the input polarization state $S_{\text{in}}$, the retardance, and the azimuthal angle of the rotating retarder.

In the first algorithm, the unknown Stokes parameters are measured by setting the direction of the rotating retarder at two orthogonal orientations $0^\circ$ and $90^\circ$, or at $45^\circ$ and $135^\circ$. For these angles, the corresponding intensities $I_{0^\circ} \sim I_{135^\circ}$ are

$$I_{0^\circ} = \frac{1}{2}\{s_0 + s_1 \cos 2\varphi + (s_2 \cos \delta + s_3 \sin \delta) \sin 2\varphi\},$$

$$I_{45^\circ} = \frac{1}{2}\{s_0 + (s_1 \cos \delta - s_3 \sin \delta) \cos 2\varphi + s_2 \sin 2\varphi\},$$

$$I_{90^\circ} = \frac{1}{2}\{s_0 + s_1 \cos 2\varphi + (s_2 \cos \delta - s_3 \sin \delta) \sin 2\varphi\},$$

$$I_{135^\circ} = \frac{1}{2}\{s_0 + (s_1 \cos \delta + s_3 \sin \delta) \cos 2\varphi + s_2 \sin 2\varphi\}.$$  

The intensities obtained by the four different micropolarizer element orientation angles $0^\circ$, $45^\circ$, $90^\circ$, and $135^\circ$ for each of the four retarder orientations indicated in Eqs. (3)–(6) are

$$I_1 = \frac{1}{2}\{s_0 + (s_2 \cos \delta + s_3 \sin \delta)\},$$

$$I_2 = \frac{1}{2}\{s_0 - s_1\},$$

$$I_3 = \frac{1}{2}\{s_0 - (s_2 \cos \delta + s_3 \sin \delta)\},$$

$$I_4 = \frac{1}{2}\{s_0 + (s_1 \cos \delta - s_3 \sin \delta)\},$$

$$I_5 = \frac{1}{2}\{s_0 + s_2\},$$

$$I_6 = \frac{1}{2}\{s_0 - (s_1 \cos \delta - s_3 \sin \delta)\},$$

$$I_7 = \frac{1}{2}\{s_0 - s_2\},$$

$$I_8 = \frac{1}{2}\{s_0 + s_1\},$$

$$I_9 = \frac{1}{2}\{s_0 + (s_2 \cos \delta - s_3 \sin \delta)\},$$

$$I_{10} = \frac{1}{2}\{s_0 - s_1\},$$

$$I_{11} = \frac{1}{2}\{s_0 - (s_2 \cos \delta - s_3 \sin \delta)\},$$

$$I_{12} = \frac{1}{2}\{s_0 + (s_1 \cos \delta + s_3 \sin \delta)\},$$

$$I_{13} = \frac{1}{2}\{s_0 + s_2\},$$

$$I_{14} = \frac{1}{2}\{s_0 - (s_1 \cos \delta + s_3 \sin \delta)\},$$

$$I_{15} = \frac{1}{2}\{s_0 - s_2\},$$

where $I_0 \sim I_3$ indicate the intensities obtained for the rotating retarder set to $0^\circ$ for each pixel polarizer element oriented at $0^\circ$, $45^\circ$, $90^\circ$, and $135^\circ$. $I_4 \sim I_7$ are the four pixel intensities obtained for the retarder oriented at $45^\circ$. $I_8 \sim I_{11}$ are likewise for the retarder at $90^\circ$, and $I_{12} \sim I_{15}$ for the retarder at $135^\circ$. Finally, the unknown input normalized Stokes parameters can be estimated from these 16 measurements by

$$\begin{bmatrix}
I_0 \\
I_1 \\
I_2 \\
I_3 \\
I_4 \\
I_5 \\
I_6 \\
I_7 \\
I_8 \\
I_9 \\
I_{10} \\
I_{11} \\
I_{12} \\
I_{13} \\
I_{14} \\
I_{15}
\end{bmatrix} = \begin{bmatrix}
1 \\
I_0 - I_1 \\
I_0 - I_2 \\
I_0 - I_3 \\
I_0 - I_4 \\
I_0 - I_5 \\
I_0 - I_6 \\
I_0 - I_7 \\
I_1 - I_2 \\
I_1 - I_3 \\
I_1 - I_4 \\
I_1 - I_5 \\
I_1 - I_6 \\
I_1 - I_7 \\
I_2 - I_3 \\
I_2 - I_4 \\
I_2 - I_5 \\
I_2 - I_6 \\
I_2 - I_7 \\
I_3 - I_4 \\
I_3 - I_5 \\
I_3 - I_6 \\
I_3 - I_7 \\
I_4 - I_5 \\
I_4 - I_6 \\
I_4 - I_7 \\
I_5 - I_6 \\
I_5 - I_7 \\
I_6 - I_7
\end{bmatrix} \begin{bmatrix}
I_0 \\
I_1 \\
I_2 \\
I_3 \\
I_4 \\
I_5 \\
I_6 \\
I_7
\end{bmatrix}.$$  

The $I_8$, $I_{10}$, $I_{13}$, and $I_{15}$ intensities are not used in Eq. (23) because they are the same intensities as other intensities. The retardance $\delta$ of the rotating retarder can be measured from the polarizer elements oriented at $0^\circ$, $45^\circ$, and $90^\circ$ with the relation

$$\delta = \cos^{-1}\left\{\frac{I_1 - I_3 + I_9 - I_{11}}{2(I_5 - I_7)}\right\}.$$  

The main drawback of this approach, which we call the “two-shot algorithm,” is that the retardance of the rotating retarder must be calibrated in advance and not dynamically because the Stokes parameters measurement use two images taken at $\xi = 0^\circ$ and $90^\circ$ or $45^\circ$ and $135^\circ$; however, for retardance measurement it is necessary to capture three images at $\xi = 0^\circ$, $45^\circ$, and $90^\circ$. When the incident polarization has $s_2 = 0$, the retardance cannot be calculated by Eq. (24). Note that if the rotating retarder employed in the system is a half-wave or a quarter-wave plate, the Stokes parameters cannot be calibrated at all via this algorithm because the Stokes parameters become undefined in that case.

In the second algorithm, the retardance of the rotating retarder can be calibrated by setting the orientation of the rotating retarder to three positions, $0^\circ$, $45^\circ$, and $90^\circ$, during measurement. In this case, the normalized Stokes parameters of the input light are obtained from
shown in Fig. 2, which shows a polarization state generator illuminated by a linear polarizer array. The extinction ratio of the micropolarizers is about 10, the spectral range is 510 to 550 nm, the pixel size is 4.65 × 4.65 µm, and the overall number of pixels is 1120 × 868.

Prior to measurement, we first calibrate the retardance of the rotating retarder using the two algorithms described above and compare the results to a measurement obtained by a commercial Mueller matrix polarimeter (Axometrics AxoScan). These results are compared in Table 1. The AxoScan measurements consist of an average of 10 measurements taken at a wavelength of 520 nm, giving the retardance of the rotating retarder as 136.6°. The proposed two-shot and three-shot calibration algorithm retardance estimates were obtained by taking the mean of nine pixels in the center of the measurement image and averaging over 10 measurements. The estimated retardance values were 135.3° and 136.7° for the two-shot and three-shot algorithms, respectively. These calibrated retardances are used in both algorithms for the Stokes parameter measurements given in the experiments below.

Next, we test the accuracy of our proposed algorithms for measuring the Stokes parameters when the polarization state generator is set to produce various polarization states. The resulting Stokes parameter estimates are shown in Table 2, obtained by extracting the spatial average of the image and a temporal average over 10 measurements. In the measurement we consider the linear diattenuation values $D_\phi$ of each micropolarizer; because the extinction ratio is low (~10) and varies from pixel to pixel, the intensity of Eq. (2) can be rewritten as

$$I = \frac{1}{2}(s_0 + D_\phi s_1 \{1 - (1 - \cos \delta)\sin^2 2\xi\} + s_2 \{1 - (1 - \cos \delta)\sin 2\delta \cos 2\xi - s_3 \sin \delta \sin 2\delta\} \cos 2\varphi) \times D_\phi \{1 - (1 - \cos \delta)\sin 2\delta \cos 2\xi\} + s_2 \{1 - (1 - \cos \delta)\cos 2\varphi + s_3 \sin \delta \cos 2\delta\} \sin 2\varphi).$$

(27)

The linear diattenuation $D_\phi$ influences the amplitude in the intensity equation [Eq. (27)]. At first, we estimated them by Fourier analysis using input rotating linear polarization images captured by only the polarization camera before measurement. Then we substitute them into Eq. (27). It indicates an absolute accuracy of 0.01 and relative precision of ±0.008, results that are comparable to conventional nonimaging (single point) Stokes polarimeter measurements [1]. The validity of the Stokes parameters measurement was confirmed in both calibration methods.

To demonstrate the advantage of dynamic calibration, we show two measurement results in which the retardance is changed by the wavelength of the incident light or by the external temperature around the polarimeter. Figure 3 shows retardance values in regard to change in wavelength from 510 to 550 nm. The photonic crystal polarization camera allows only horizontal polarization to be captured by only the polarization camera before measurement. For this experiment, the incident polarization is 22.5° linear polarization. As expected the retardances exhibit a linear decrease with wavelength.

The second measurement (Fig. 4) shows retardance changing with temperature from 20°C to 30°C. By the dynamic calibration, the calibrated retardances show a linear variation over the temperature range. The results of Stokes parameters measurement and retardance between the two-shot (static) and the three-shot (dynamic) methods for right circular polarization is shown in

**Table 1. Comparing Retardance Estimates by the Two Proposed Algorithms and the AxoScan**

<table>
<thead>
<tr>
<th>Methods</th>
<th>Mean</th>
<th>Repeatability Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>AxoScan</td>
<td>136.6</td>
<td>±0.06</td>
</tr>
<tr>
<td>Two-shot</td>
<td>135.3</td>
<td>±0.34</td>
</tr>
<tr>
<td>Three-shot</td>
<td>136.7</td>
<td>±0.24</td>
</tr>
</tbody>
</table>

**Table 2. Accuracy of Results for Stokes Parameter Measurements by Rotating Retarder and Polarization Cameras**

<table>
<thead>
<tr>
<th>Stokes Parameters</th>
<th>Polarization State</th>
<th>Theoretical Value</th>
<th>Two-Shot</th>
<th>Three-Shot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal linear</td>
<td>1</td>
<td>0.99 ± 0.007</td>
<td>0.99 ± 0.007</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.01 ± 0.008</td>
<td>0.01 ± 0.008</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.00 ± 0.005</td>
<td>0.00 ± 0.005</td>
<td></td>
</tr>
<tr>
<td></td>
<td>−45° linear</td>
<td>0</td>
<td>0.01 ± 0.001</td>
<td>0.00 ± 0.001</td>
</tr>
<tr>
<td></td>
<td>−1</td>
<td>−0.99 ± 0.008</td>
<td>−0.99 ± 0.007</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.01 ± 0.007</td>
<td>0.00 ± 0.008</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Right circular</td>
<td>1</td>
<td>0.00 ± 0.007</td>
<td>0.00 ± 0.007</td>
</tr>
<tr>
<td></td>
<td>polarization</td>
<td>0</td>
<td>0.01 ± 0.002</td>
<td>−0.01 ± 0.007</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.99 ± 0.006</td>
<td>0.99 ± 0.004</td>
<td></td>
</tr>
</tbody>
</table>
Table 3. The two-shot method exhibits errors because it cannot recalibrate the retardance. However, the three-shot method is not influenced by retardance error caused by the temperature change.

The strong point of working with a polarization camera is its ability to image dynamic variation of polarization, and our calibration method adapts this instrument so that it can easily measure circular polarization and not just linear polarization. To demonstrate our method’s ability to measure dynamic phenomena, we present a third experiment, showing the polarization change due to time-varying stress-induced birefringence. A finger presses down on a U-shaped film of polyethylene terephthalate (PET) plastic, as shown in Fig. 5. Using the two-shot algorithm, we measured the dynamic Stokes parameters of the light transmitted by the U-shaped film. In this case, the Stokes parameters were measured at a rate of 10 Hz by synchronizing the rotating retarder with the polarization camera readout. For this measurement, the incident polarization state is right circular polarization to allow for birefringence measurement that is independent of orientation. The resulting time-resolved Stokes parameters are shown in Fig. 5, where we can see the dynamic changes in the Stokes parameters due to stress induced in a U-shaped film from pressing down on an edge with a finger.

In conclusion, we have shown two methods to calibrate and perform complete Stokes polarimetry with a rotating retarder in front of a polarization camera employing a pixelated micropolarizer. While both methods allow dynamic capture of full Stokes parameter images, one method requires calibration in advance while another allows dynamic recalibration during measurement. The dynamic recalibration method can be useful for measuring outdoors over long time periods. This capability can be of particular advantage for imagers such as microbolometer infrared cameras that experience rapid changes in detector calibration due to drift and pixel response nonuniformity.

REFERENCES


Table 3. Comparison of Polarization Measurements Between the Two-Shot and the Three-Shot Methods

<table>
<thead>
<tr>
<th>Room Temperature [°C]</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed retardance [°]</td>
<td>135.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two-shot</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stokes parameters</td>
<td>1</td>
<td>-0.06</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>0.96</td>
<td>0.99</td>
<td>0.93</td>
</tr>
<tr>
<td>Retardance [°]</td>
<td>137.2</td>
<td>135.7</td>
<td>132.1</td>
</tr>
<tr>
<td>Three-shot</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stokes parameters</td>
<td>1</td>
<td>0.02</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>1.01</td>
<td>0.99</td>
<td>1.02</td>
</tr>
</tbody>
</table>

Fig. 3. Retardance of the rotating retarder changing wavelength. The black line gives the theoretical retardance, while the red circles are measurements from our proposed system.

Fig. 4. Retardance of the rotating retarder changing temperature. A black line is measurement by the Axoscan, the red points are measurements by our instrument.

Fig. 5. Dynamic Stokes parameter images of stressed U-shaped PET plastic film (Visualization 1).